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A Method for Calculation of Fastener Torque Specifications Which Includes Statistical Tolerancing

ABSTRACT: The Monte Carlo simulation method is applied to quantify the probability of exceeding the maximum desired bolt exploitation for fasteners tightened to a torque specification. The simulation results are utilized to develop a method for the calculation of torque specifications such that the probability that a bolt will exceed the maximum desired bolt exploitation is thirty-two parts per million (ppm).

KEYWORDS: torque, friction, fastener, calculate

Nomenclature

The following is a list of the nomenclature used. Most terms are consistent with the nomenclature used in the VDI procedure for Systematic Calculation of High Duty Bolted Joints (VDI 2230).

A_0	Smallest cross-section area of bolt
A_S	Effective tensile stress cross-section of the bolt thread per ISO 898-1
D_0	Outside diameter of bolt at the smallest cross-section, A_0 (smaller of D_S or D_T)
D_2	Pitch diameter of bolt thread
D_3	Minor diameter of bolt thread
D_{km}	Effective diameter for friction at the contact of the head of the driven fastener
D_S	Diameter at stress cross-section A_S
D_T	Shank diameter of bolt neck
D_W	Outside diameter of the contact area under the head of the driven fastener
F_M	Assembly preload, bolt tensile load at tightening
$F_{M,\nu}$	Allowable bolt preload, bolt tensile load at which the equivalent stress considering tension and torsion is $\nu R_{p,0.2}$
$F_{M,MIN}$	Minimum assembly preload expected from tightening to the specified torque
$F_{M,MAX}$	Maximum assembly preload expected from tightening to the specified torque
M_A	Assembly input torque
$M_{A,MIN}$	Maximum assembly input torque
$M_{A,MAX}$	Minimum assembly input torque
$M_{A,PRE}$	Assembly prevailing torque
M_G	Assembly thread torque, moment in the bolt neck
P	Pitch of the bolt thread
R_y	Actual proof stress for a bolt
$R_{p,0.2}$	Minimum 0.2 % proof stress of bolt material per ISO 898-1

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d_i	Inside diameter of hollow bolt
d_h	Inside diameter of the contact area under the head of the driven fastener
β_{th}	Half flank angle of the bolt thread ($\pi/6$ for ISO thread)
μ_G	Coefficient of friction between bolt and nut thread
$\mu_{G,MIN}$	Minimum coefficient of friction between bolt and nut thread
$\mu_{G,MAX}$	Maximum coefficient of friction between bolt and nut thread
μ_K	Coefficient of friction at the contact of the driven fastener head
$\mu_{K,MIN}$	Minimum coefficient of friction at the contact of the driven fastener head
$\mu_{K,MAX}$	Maximum coefficient of friction at the contact of the driven fastener head
ν	Degree (%) of exploitation of bolt yield stress desired at maximum assembly condition

Introduction

In order to minimize product cost and mass, bolted attachments should be designed with the smallest fasteners possible. This is achieved in part by designing a torque specification that results in the maximum possible preload, but with acceptable margin against failure due to over-stressing during tightening or in service. To accomplish this, the probability density function for the ratio of bolt preload to the targeted allowable bolt preload must be determined. This is not a simple task because the ratio is a non-linear function of random variables. The bolt preload variation is related to the variation in input torque, driven fastener head friction, and fastener thread friction. At the same time the variation in the targeted allowable preload is related to the variation in bolt yield strength and fastener thread friction.

A method for the calculation of assembly torque specifications is developed for the general case, which includes bolts with reduced shank diameter, bolts that are hollow, and bolts or nuts with prevailing torque. The method includes the calculation of torque specification limits based on the simultaneous occurrence of extreme values of process variables, and it is referred to as the Extreme Value Method. The upper torque specification limit is calculated so that the bolt preload is equal to the desired maximum value when the extreme values of process variables occur. The lower torque specification limit is calculated so that the assembly process will be statistically capable for the selected assembly tool.

The Monte Carlo computer simulation method is then applied to simulate the tightening process with normally distributed process variables and to calculate the non-normal distribution that describes the probability of exceeding the maximum desired bolt exploitation. The simulation results are used to modify the Extreme Value Method so that the probability that a bolt will exceed the maximum desired bolt exploitation is 32 ppm.

The Extreme Value Method

For known values of bolt geometry, bolt material properties, prevailing torque, and fastener head and thread friction, the Extreme Value Method is used to calculate the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) and the bolt preload limits ($F_{M,MIN}$ and $F_{M,MAX}$). The bolt preload limits are used to calculate the capability of the fastened joint to resist the loads expected in service.

First, the equation for maximum allowable initial bolt preload ($F_{M,MAX}$) is developed so that the equivalent stress in the outer fiber of the bolt due to tensile and torsional loads at the minimum section is at the desired maximum allowable stress ($\nu R_{P,0.2}$). The exploitation factor (ν) is a factor to provide a margin against yielding the bolt during tightening to account for an expected increase in bolt load during service or to account for possible error in the estimation of the limiting values of friction. The maximum allowable bolt preload is at the most conservative value when the bolt has minimum yield strength ($R_{P,0.2}$) and minimum thread friction ($\mu_{G,MIN}$).

$$F_{M,MAX} = -6M_{A,PRE}K_1K_2 + \frac{\sqrt{\left(6M_{A,PRE}K_1K_2\right)^2 - 4\left(1+3K_1K_2^{\,2}\right)\left(3M_{A,PRE}^{\,2}K_1 - K_3\right)}}{2\left(1+3K_1K_2^{\,2}\right)} \tag{1}$$

Where:

$$K_1 = \left[\frac{4D_0}{\left(D_0^{\,2} + d_i^{\,2}\right)}\right]^2, \quad K_2 = \left[\frac{P}{2\pi} + \frac{\mu_{G,MIN}D_2}{2\cos\beta_{th}}\right], \text{ and } K_3 = \left[\frac{\nu R_{P,0.2}\pi\left(D_0^{\,2} - d_i^{\,2}\right)}{4}\right]^2$$

For the case where the minimum bolt cross section is the threaded section:

$$D_0 = \frac{D_2 + D_3}{2} \tag{2}$$

And for the case where the minimum bolt cross section is the shank:

$$D_0 = D_T \tag{3}$$

The mathematical model of the tightening process developed by Motosh [2] is modified here to include the prevailing torque term:

$$F_M = \frac{\left(M_A - M_{A,PRE}\right)}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right)} \tag{4}$$

Where:

$$D_{km} = \frac{D_w + d_h}{2} \tag{5}$$

This equation is rearranged to solve for input torque:

$$M_A = M_{A,PRE} + F_M\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right) \tag{6}$$

The maximum torque limit is calculated by substituting the maximum allowable initial preload ($F_{M,MAX}$) from Eq 1, the minimum value of head friction ($\mu_{K,MIN}$), and the minimum value of thread friction ($\mu_{G,MIN}$) into Eq 6. This maximum torque results in the maximum allowable initial preload in the worst-case conditions for friction with respect to increasing preload.

$$M_{A,MAX} = M_{A,PRE} + F_{M,MAX}\left[\frac{P}{2\pi} + \frac{\mu_{G,MIN} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MIN} D_{km}}{2}\right] \tag{7}$$

The minimum torque limit is calculated from the maximum torque based on the desired torque tolerance:

$$M_{A,MIN} = M_{A,MAX}\frac{\left(1 - M_{A,TOL\%}\right)}{\left(1 + M_{A,TOL\%}\right)} \tag{8}$$

Where:

$$M_{A,TOL\%} = \frac{M_{A,TOL}}{M_{A,NOM}} \tag{9}$$

$$M_{A,TOL} = \frac{\left(M_{A,MAX} - M_{A,MIN}\right)}{2} \tag{10}$$

Finally, the bolt minimum initial preload when assembled to the minimum torque is calculated by substituting $M_{A,MIN}$ into Eq 4 with maximum values of head and thread friction:

$$F_{M,MIN} = \frac{\left(M_{A,MIN} - M_{A,PRE}\right)}{\frac{P}{2\pi} + \frac{\mu_{G,MAX} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MAX} D_{km}}{2}} \tag{11}$$

The application of these equations requires that the coefficients of head and thread friction be calculated with equations that also include the consideration of prevailing torque. The equations are:

$$\mu_G = \frac{2\cos\beta_{th}}{D_2}\left(\frac{M_G - M_{A,PRE}}{F_M} - \frac{P}{2\pi}\right) \tag{12}$$

And,

$$\mu_K = \frac{2\left(M_A - M_G\right)}{D_{km} F_M} \tag{13}$$

In summary, the Extreme Value Method is applied as follows:

1. Laboratory tests are performed. Multiple samples of the attachment are tightened while the input torque, bolt tension, and thread torque are measured. The coefficients of head friction and coefficient of thread friction are calculated for each sample with Eqs 12 and 13. The statistics, mean and standard deviation, for the head and thread friction are calculated from the individual samples.

2. The values $\mu_{K,MIN}$, $\mu_{K,MAX}$, $\mu_{G,MIN}$, and $\mu_{G,MAX}$, are calculated as the three standard deviation limits of the individual friction values calculated above.
3. The required torque specification tolerance $M_{A,TOL\ \%}$ is determined so that the desired assembly tool will be process capable.
4. The maximum desired degree of exploitation of the bolt during the tightening process (ν) is selected.
5. Equations 1, 7, 8, and 11 are applied to calculate the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) and the bolt initial preload limits ($F_{M,MIN}$ and $F_{M,MAX}$).

The Extreme Value Method (Simple Case)

For the case where prevailing torque is zero, the bolt is solid, and the shank is not reduced. The equations above reduce to the following set of equations. Equation 14 is the same as derived in the procedure VDI 2230.

$$F_{M,\nu} = \frac{A_S \nu R_y}{\sqrt{1+3\left[\frac{4}{1+D_3/D_2}\left(\frac{P}{\pi D_2}+\frac{\mu_G}{\cos\beta_{th}}\right)\right]^2}} \tag{14}$$

$$F_{M,MAX} = \frac{A_S \nu R_{P,0.2}}{\sqrt{1+3\left[\frac{4}{1+D_3/D_2}\left(\frac{P}{\pi D_2}+\frac{\mu_{G,MIN}}{\cos\beta_{th}}\right)\right]^2}} \tag{15}$$

$$F_M = \frac{M_A}{\left(\frac{P}{2\pi}+\frac{\mu_G D_2}{2\cos\beta_{th}}+\frac{\mu_K D_{km}}{2}\right)} \tag{16}$$

Where:

$$D_{km} = \frac{D_w + d_h}{2} \tag{17}$$

$$M_{A,MAX} = F_{M,MAX}\left[\frac{P}{2\pi}+\frac{\mu_{G,MIN} D_2}{2\cos\beta_{th}}+\frac{\mu_{K,MIN} D_{km}}{2}\right] \tag{18}$$

$$M_{A,MIN} = M_{A,MAX}\frac{\left(1-M_{A,TOL\%}\right)}{\left(1+M_{A,TOL\%}\right)} \tag{19}$$

Where:

$$M_{A,TOL\%} = \frac{M_{A,TOL}}{M_{A,NOM}} \tag{20}$$

$$M_{A,TOL} = \frac{\left(M_{A,MAX}-M_{A,MIN}\right)}{2} \tag{21}$$

$$F_{M,MIN} = \frac{M_{A,MIN}}{\frac{P}{2\pi} + \frac{\mu_{G,MAX} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MAX} D_{km}}{2}} \tag{22}$$

The equations for the coefficients of head and thread friction reduce to the more standard equations as defined in the ISO standard for Fasteners – Torque/Clamp Force Testing (16047) and in the German national standard, Determination of Coefficient of Friction of Bolt/Nut Assemblies Under Specified Conditions (DIN 946).

$$\mu_G = \frac{2\cos\beta_{th}}{D_2}\left(\frac{M_G}{F_M} - \frac{P}{2\pi}\right) \tag{23}$$

And:

$$\mu_K = \frac{2(M_A - M_G)}{D_{km} F_M} \tag{24}$$

Statistical Analysis

The probability that the extreme values of the process variables ($M_{A,MAX}$, $\mu_{K,MIN}$, $\mu_{G,MIN}$, and $R_{P,0.2}$) will occur simultaneously is very small, and therefore the Extreme Value Method is very conservative. The application of statistical tolerancing methods results in an increase of the torque specification limits and therefore bolt preload, with an acceptable probability that over-loading of the bolt will occur. Equations for calculation of the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) are developed so that the probability is approximately 32 ppm that the ratio of the actual bolt preload to the allowable bolt preload (F_M / $F_{M,\nu}$) will exceed the exploitation factor (ν). This is equal to a four-standard-deviation (4σ) probability for a normal distribution. Also, equations for the calculation of the bolt preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) are developed so that the probability is 0.0014 (on each side) that the bolt preload will be outside of those limits. This is equal to a three-standard-deviation (3σ) probability for a normal distribution.

A common analytical method used to determine the density function of a complex transfer function is the *error transmission formula* [2], which is a Taylor's series expansion technique. The application of this method results in a formula for the standard deviation of the bolt preload that is a function of the standard deviations of M_A, μ_K, and μ_G.

$$\sigma_{F_M}{}^2 = \left(\frac{\partial F_M}{\partial M_A}\right)^2 \sigma_{M_A}{}^2 + \left(\frac{\partial F_M}{\partial \mu_K}\right)^2 \sigma_{\mu_K}{}^2 + \left(\frac{\partial F_M}{\partial \mu_G}\right)^2 \sigma_{\mu_G}{}^2 \tag{25}$$

However, two assumptions made in the derivation of the *error transmission formula* do not apply for this problem:

1. The random variables are assumed to be independent of each other. This does not apply in this case since the head friction and thread friction are sometimes dependent on each other, in the case where the bolt is the driven fastener.
2. The higher order terms of the Taylor's series expansion are assumed insignificant and are excluded. In this case, the bolt preload is sufficiently non-linear within the normal

tolerances of the process variables, and the higher-order terms of the Taylor's series expansion are significant.

Because of the complexity of the transfer functions and the dependencies between variables, a Monte Carlo computer simulation is chosen to model the tightening process and calculate the probability functions needed. The computer uses a random number generator to draw values from the distributions of the process variables (R_y, M_A, μ_K, and μ_G) and computes the values for F_M and $F_{M,v}$. The computed values are then summarized in histograms for analysis. Various distributions of the process variables are simulated to allow for a generalization of results. The simulation procedure for each test case is as follows:

1. The bolt geometry properties (P, D_T, D_2, D_3, d_i, d_h, A_s, D_{km}, and β_{th}) are modeled as constants. Values are selected from the appropriate ISO specifications.
2. Values for head friction (μ_K) and for thread friction (μ_G) are randomly generated to fit normal distributions. The means and standard deviations are selected for each test case, and for some cases the thread friction is generated to be directly dependent on head friction.
3. Values of torque (M_A) are randomly generated to fit a normal distribution with the three standard deviation limits equal to $M_{A,MIN}$ and $M_{A,MAX}$. The values for $M_{A,MIN}$ and $M_{A,MAX}$ are calculated from the appropriate equations.
4. Values of bolt yield strength (R_y) are randomly generated to fit a normal distribution so that $R_{p,0.2}$ is the lower three-standard deviation limit and the upper three-standard deviation limit is $R_{p,0.2}$ times the ratio of maximum to minimum specified hardness for the bolt material.
5. All variation is statistically independent except for the variation in μ_K and μ_G as stated above.
6. The maximum allowable bolt preload ($F_{M,v}$) is calculated for each simulated sample with values of R_y and μ_G randomly generated.
7. The bolt initial preload (F_M) is calculated for each simulated sample with values of M_A, μ_K, and μ_G randomly generated.
8. The ratio $F_M/F_{M,v}$ is calculated for each simulated sample.

Example Simulation Result for the Extreme Value Method

The example model is a standard M10 bolt per ISO 4162 with grade 10.9 material per ISO 898-1. The statistics for the head and thread friction are values for a commonly used organic finish, and the torque specification limits are calculated using the Extreme Value Method. The results are shown in the first column of Table 1, labeled Extreme Value Method (EVM), and in Figs. 1 and 2. Results are shown for cumulative probability values that are equivalent to three standard deviations (3σ) for a normal distribution (0.0014 and 0.9986) and for cumulative probability values that are equivalent to four standard deviations (4σ) for a normal distribution (0.000032 and 0.999968). Here are some summary comments from the example simulation.

1. The probability that the bolt preload will be outside of the preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) is significantly lower than the desired probability.

2. The probability is about 32 ppm that the ratio $F_M / F_{M,\nu}$ exceeds 92.5 %. The desired result is that if the probability is about 32 ppm, the ratio $F_M / F_{M,\nu}$ exceeds 100 %, since the exploitation factor for this example is 1.0.
3. The distributions of F_M and the ratio $F_M / F_{M,\nu}$ are not normal since the distribution is not symmetrical about the mean value. This is proof that the *error transmission formula* should not be applied.
4. As expected, the results show that the Extreme Value Method is over conservative.

Summary comments from other simulations of the Extreme Value Method are as follows:

1. For the example shown above, the values of head friction and thread friction are dependent. In the case where the values are independent, the probability that the bolt preload will be greater than $F_{M,MAX}$ or less than $F_{M,MIN}$ is decreased. However, the probability that the bolt will yield is approximately the same as above.
2. In the case where the variation for head and thread friction are increased by 50 %, the probability that the bolt preload will be greater than $F_{M,MAX}$ or less than $F_{M,MIN}$ is also decreased. However, the probability that the bolt will yield is again approximately the same.

TABLE 1—*Simulation results for an M10 bolt.*

			Extreme Value Method	Statistical Method
Inputs				
ν			100 %	100 %
$\mu_{G,nom}$			0.13	0.13
$\mu_{G,std\ dev}$			0.01	0.01
$\mu_{K,nom}$			0.13	0.13
$\mu_{K,std\ dev}$			0.01	0.01
Calculations				
$M_{A,MIN}$ (Nm)			51.4	55.6
$M_{A,MAX}$ (Nm)			69.6	75.2
$F_{M,MIN}$ (KN)			22.8	27.2
$F_{M,NOM}$ (KN)			32.3	34.9
$F_{M,MAX}$ (KN)			46.5	45.2
Simulation Results				
F_M	(KN)	(min sample)	23.1	25.3
F_M	(KN)	(Prob. = 0.000032)	22.9	25.0
F_M	(KN)	(Prob. = 0.0014)	25.3	27.5
F_M	(KN)	(Prob. = 0.5000)	32.4	35.0
F_M	(KN)	(Prob. = 0.9986)	41.9	45.6
F_M	(KN)	(Prob. = 0.999968)	45.1	49.1
F_M	(KN)	(max sample)	46.1	50.7
$F_M / F_{M,\nu}$		(Prob. = 0.9986)	85.0 %	92.5 %
$F_M / F_{M,\nu}$		(Prob. = 0.999968)	92.5 %	100.2 %
$F_M / F_{M,\nu}$		(max sample)	92.4 %	101.7 %
$F_M / F_{M,MIN}$		(min sample)	101.3 %	92.3 %
$F_M / F_{M,MIN}$		(Prob. = 0.000032)	100.4 %	91.2 %
$F_M / F_{M,MIN}$		(Prob. = 0.0014)	111.0 %	100.4 %
$F_M / F_{M,MAX}$		(Prob. = 0.9986)	90.1 %	100.1 %
$F_M / F_{M,MAX}$		(Prob. = 0.999968)	97.0 %	108.6 %
$F_M / F_{M,MAX}$		(max sample)	99.1 %	112.2 %

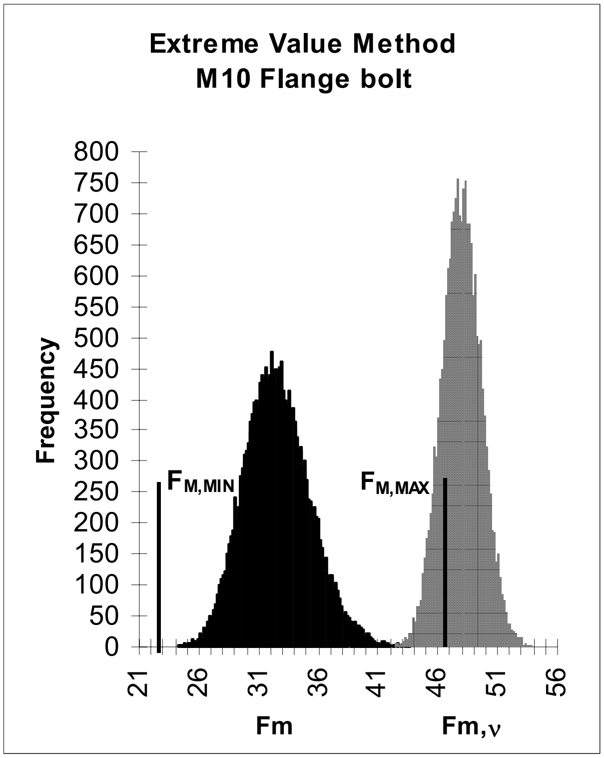

FIG. 1—*Distribution of* F_M *and* $F_{M,\nu}$ *for the EVM.*

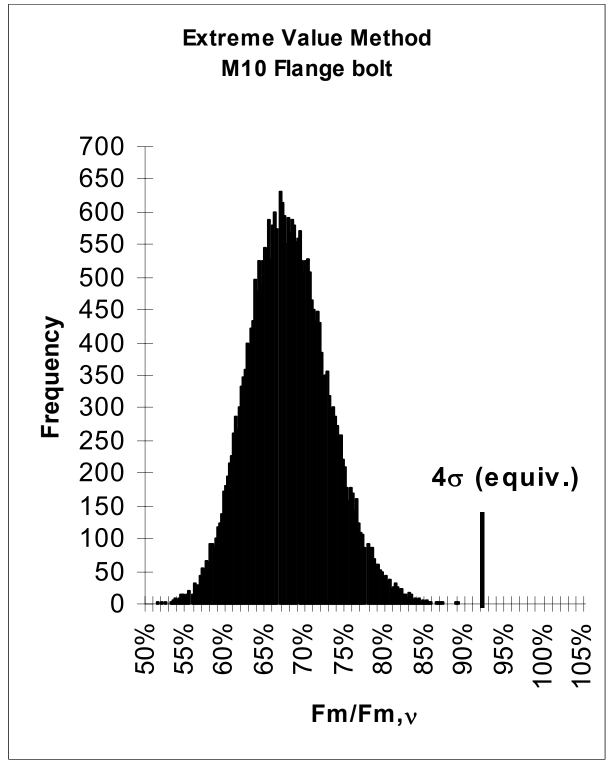

FIG. 2—*Distribution of* $F_M / F_{M,\nu}$ *for the EVM.*

Statistical Targeting

Statistical targeting of the torque specification is accomplished by modification of the equations based on the results from the Monte Carlo simulations. The goals as stated above are:

1. To develop equations for calculation of the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) such that the probability is approximately 32 ppm that the ratio of the bolt preload to the allowable bolt preload ($F_M / F_{M,\nu}$.) will exceed the exploitation factor (ν).
2. To develop equations for the calculation of the bolt preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) such that the probability is 0.0014 (each side) that the bolt preload will be outside of those limits.

The revised equations are shown here for the simple case where there is no prevailing torque and the bolt is solid. Modifications are placed in brackets {} to be seen easily.

$$F_{M,\nu} = \frac{A_S \nu R_y}{\sqrt{1+3\left[\frac{4}{1+D_3/D_2}\left(\frac{P}{\pi D_2}+\frac{\mu_G}{\cos\beta_{th}}\right)\right]^2}} \qquad (26)$$

$$F_{M,4\sigma} = \frac{A_S \nu R_{P,0.2} / \{0.925\}}{\sqrt{1 + 3\left[\frac{4}{1 + D_3/D_2}\left(\frac{P}{\pi D_2} + \frac{\mu_{G,MIN}}{\cos\beta_{th}}\right)\right]^2}} \tag{27}$$

$$F_M = \frac{M_A}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2\cos\beta_{th}} + \frac{\mu_K D_{km}}{2}\right)} \tag{28}$$

Where:

$$D_{km} = \frac{D_w + d_h}{2} \tag{29}$$

$$M_{A,MAX} = \{F_{M,4\sigma}\}\left[\frac{P}{2\pi} + \frac{\mu_{G,MIN} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MIN} D_{km}}{2}\right] \tag{30}$$

$$M_{A,MIN} = M_{A,MAX} \frac{\left(1 - M_{A,TOL\%}\right)}{\left(1 + M_{A,TOL\%}\right)} \tag{31}$$

Where:

$$M_{A,TOL\%} = \frac{M_{A,TOL}}{M_{A,NOM}} \tag{32}$$

$$M_{A,TOL} = \frac{\left(M_{A,MAX} - M_{A,MIN}\right)}{2} \tag{33}$$

$$F_{M,MAX} = \frac{\{0.9\} M_{A,MAX}}{\frac{P}{2\pi} + \frac{\mu_{G,MIN} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MIN} D_{km}}{2}} \tag{34}$$

$$F_{M,MIN} = \frac{\{111.0\} M_{A,MIN}}{\frac{P}{2\pi} + \frac{\mu_{G,MAX} D_2}{2\cos\beta_{th}} + \frac{\mu_{K,MAX} D_{km}}{2}} \tag{35}$$

$$\mu_G = \frac{2\cos\beta_{th}}{D_2}\left(\frac{M_G}{F_M} - \frac{P}{2\pi}\right) \tag{36}$$

$$\mu_K = \frac{2\left(M_A - M_G\right)}{D_{km} F_M} \tag{37}$$

The example above is simulated again, but this time with the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) and the bolt preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) calculated using Eqs 30, 31, 34, and 35. The results are shown in the second column of Table 1 and in Figs. 3 and 4:

1. The probability that the bolt preload will be outside of the preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) is approximately equivalent to 3σ limits for a normal distribution. This is the desired result.
2. The probability is equivalent to 4σ limits (32 PPM) that the ratio $F_M / F_{M,\nu}$ exceeds 100 %. This is the desired result.

Considerations for Setting Exploitation Factor

Normally, the exploitation factor (ν) is a factor to provide for a margin against yielding the bolt during tightening to account for an expected increase in bolt load during service. A reduction in the exploitation factor should also be considered in order to compensate for inaccurate estimates of the limiting values of head and thread friction, for example, if a small sample size is used in the laboratory test.

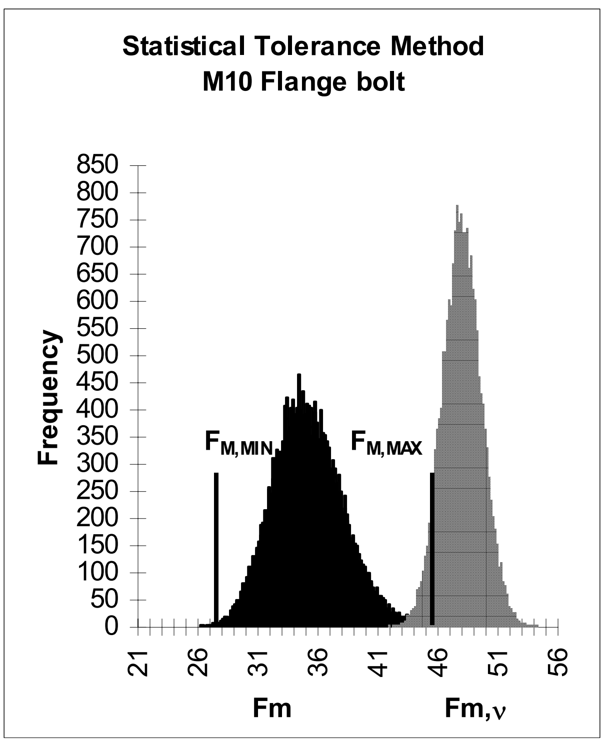

FIG. 3—*Distribution of F_M and $F_{M,\nu}$ for the EVM.*

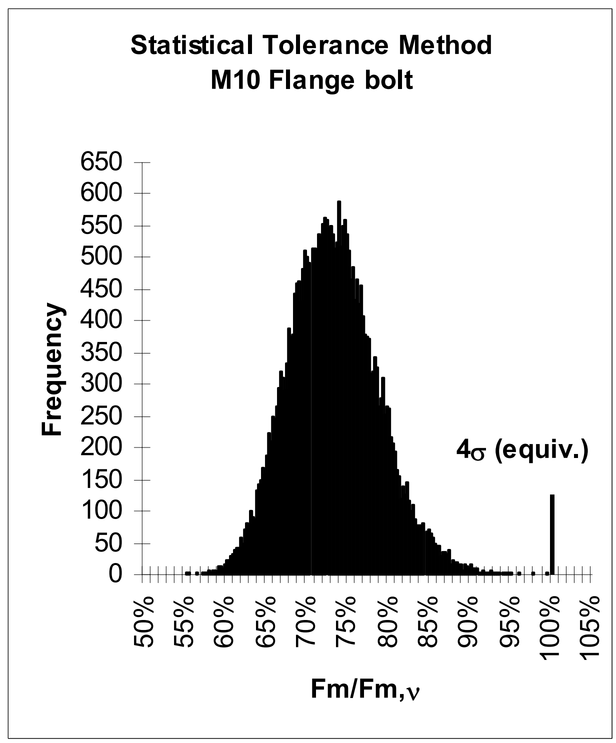

FIG. 4—*Distribution of $F_M / F_{M,\nu}$ for the EVM.*

Conclusion

For known values of bolt geometry, bolt material properties, prevailing torque, and fastener head and thread friction, the Extreme Value Method calculates the torque specification limits ($M_{A,MIN}$ and $M_{A,MAX}$) and the bolt preload limits ($F_{M,MIN}$ and $F_{M,MAX}$) so that the probability is approximately 32 ppm (equivalent to 4σ for a normal distribution) that the bolt equivalent stress will exceed the desired maximum allowable stress ($\nu R_{P,0.2}$). In other words, if ν is set to 100 %, then the probability is approximately 32 ppm that the equivalent stress in the bolt will exceed the minimum yield stress $R_{P,0.2}$. This is quite conservative since few bolts will actually have yield strength at the minimum specified value.

Application of statistical tolerancing methods results in a modification to the Extreme Value Method. Equations 26–37 describe the method for the simple case where there is no prevailing torque and the bolt is solid. The result is an increase to the calculated torque specification and an increase in overall bolt preload, so that the probability is approximately 32 ppm (equivalent to 4σ for a normal distribution) that the bolt equivalent stress will exceed the desired maximum allowable stress (νR_y). In the case where ν is set to 100 %, the probability is approximately 32 ppm that the bolt will yield, with the actual distribution of bolt yield strength considered. The result is an 8 % increase in average bolt tension with a very small risk of yielding a bolt during assembly.

References

[1] Motosh, N., "Development of Design Charts for Bolts Preloaded up to the Plastic Range," *J. Eng. Ind.*, August 1976.
[2] Cramer, H., "Mathematical Methods for Statistics," Princeton University Press, 1942.